Experimental Research Regarding Viscous Friction on Rotating Disks

ADRIAN CIOCĂNEA[1], MARIANA FLOREA[2], NICOLAE BĂRAN[1]*, GHEORGHE BĂRAN[3]
[1] Politehnica University of Bucharest, 313 Splaiul Independenþei, 060042, Bucharest, Romania
[2] Politehnica University of Bucharest, Department of Hydraulics and Hydraulics Machines, 313 Splaiul Independenþei, 060042, Bucharest, Romania
[3]INCDIE ICPE-CA Bucharest, 313 Splaiul Independenþei, 060042, Bucharest, Romania

The paper presents the results of the experimental researches regarding the establishment of the viscous friction coefficient in the case of a circular disk rotating inside a vessel.Six working liquids were used in order to obtain a wide range of values for the viscous friction coefficient in function of the Reynolds number ($10^2 \div 5 \cdot 10^5$).

Keywords: viscous friction, friction moment on disk

The computation of the power lost by viscous friction on disks with uniform rotation is a classic problem [6]. It can be found in specialized literature [3], [4], [10] and constitutes also the main subject of several PhD theses [7], [9]. Every edition presents different experimental results as in [4] and [3], [6]. Beside this, new research regarding the problem is performed (like NACA [3]) and new topics arise, like flow regime stability [2]. The problem is applied especially in turbo-machines field, in different industrial plants, in some process equipment [5] a.s.o.

The friction moment on the disk, C_M has the following expression [3] [6]:

$$C_M = \frac{M}{0.5\rho\omega^2 R^5}, \tag{1}$$

where:

M – friction moment;
ρ - fluid density;
$\omega = \pi n / 30$ - angular speed ;
R - disk radius.
C_M depends on the Reynolds number

$$\mathrm{Re} = \omega R^2 / \nu \tag{2}$$

on the flow regime (laminar or turbulent), with $\mathrm{Re}_{cr} = 3 \cdot 10^5$ [3, 4], on the surface roughness and the distance *s* between the disk and the bottom of the vessel; ν represents the cinematic viscosity.

Theoretical results

Theoretically, the problem has been approached for the first time by Kàrman [3, 4, 10] for a disk with uniform rotation in viscous, incompressible and unlimited fluid. Integrating the Navier-Stokes equations and assuming permanent movement with axial symmetry, Kàrman (1921) obtains analytically the following expression:

$$C_M = 4,7 / \sqrt{\mathrm{Re}}, \tag{3}$$

improved by Cochran (1934), which changes the limit condition and uses numerical integration

$$C_M = 3,87 / \sqrt{\mathrm{Re}}. \tag{4}$$

Both relations are valid in laminar regime ($\mathrm{Re} < 3 \cdot 10^5$). Kàrman introduces also the relation for turbulent regime [3, 4]:

$$C_M = 0,146 / (\mathrm{Re})^{1/5}, \quad 3 \cdot 10^5 < \mathrm{Re} < 10^6. \tag{5}$$

For $10^5 < \mathrm{Re} < 2 \cdot 10^6$ Goldstein (1935) infers an implicit formula [3]:

$$1/\sqrt{C_M} = 1,87 \log\left(\mathrm{Re}\sqrt{C_M}\right) + 0,03, \tag{6}$$

slightly changed in a previous edition [4], presented also in [3]:

$$1/\sqrt{C_M} = \left(1/K\sqrt{8}\right)\ln\left(\mathrm{Re}\sqrt{C_M}\right) + 0,03 \tag{7}$$

with K = 0,41 - Kàrman's constant.

For cased disks the following relations are given [3-5]:

$$C_M = 2,67 / \sqrt{\mathrm{Re}}, \quad 10^4 < \mathrm{Re} < 3 \cdot 10^5 \tag{8}$$

$$C_M = 0,062 / \sqrt[5]{\mathrm{Re}}, \quad \mathrm{Re} > 3 \cdot 10^5 \tag{9}$$

for laminar, respectively turbulent regime. If the disks are cased, but the *s* distances are small, the Couette flows applies and the viscous friction effort and the friction torque have the following expression

$$\tau = \eta \frac{r\omega}{s}; \quad M = 2\pi \int_0^R \tau r^2 dr = \frac{\pi}{2} \frac{\omega \eta R^4}{s}.$$

For both disks faces the coefficient will be

$$C_M = 2\pi (R/s)(1/\mathrm{Re}), \quad \mathrm{Re} << \mathrm{Re}_{cr} \tag{10}$$

The synthesis of the presented results (5) ÷ (10) can be seen in the diagram of figure 1 [3].

The measurements performed by Kempf (1924), Schmidt (1921), Riabouchinsky (1951) and recently by NACA in subsonic and supersonic regime verifies more or less the theoretical results. Beside that, in [6] is shown that the case has a significant influence for *s*/*R* < 0,3. Several empiric formulas, synthesized in [1], are published for smooth or rough disks.

* *email: n_baran_fimm@yahoo.com; Tel.: (+40) 021 318 10 19*

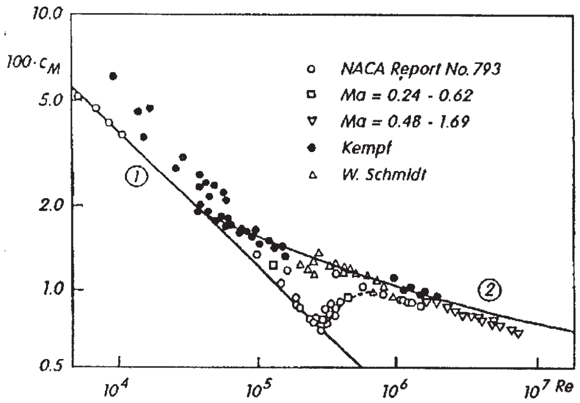

Fig. 1. Theoretical dependency CM(Re) and experimental validation 1 - equation (4), 2-equation (7)

Experimental part

Looking at the theoretical and the experimental results it can be seen that the research is still in process. The older experimental research (Kempf, Schmidt), likewise newer ones (NACA), confirms more or less theoretical results (fig. 2). Therefore, the outcomes regarding C_M at enclosed disks (s/R = 0,03) using air as work fluid, are approximately 17% higher than theoretical ones [4].

The test unit (fig. 2) consists in a thin disk (2 mm) with Φ100 mm mounted in a cylindrical case, 100 mm high (s/R = 1). The disk is rotated by d.c. motor having n = 300 ÷ 1400 rpm.

First the motor had worked with air as working liquid. The power consumption in this regime $P_g(n)$ was thus determined. Then, with the mechanical power P_m connected to the shaft, the diagrams P_m=f(n) at constant voltage U = 3 ÷ 8 V were obtained. The power consumed because of viscous friction on disk was computed as $P=P_m-P_g=M\omega$.

It has been noticed that, during measurements, temperature has raised with only 1°C.

With $Re=\omega^2R / \nu, Re_{cr} = 3 . 105$, $\omega=\pi n / 30$ rad/s for the laminar regime can be obtained $\omega R^2 / \nu<3 . 10^5$; $\nu >8,722 . 10^{-9}$ m²/s, respectively

$$\nu > (0,0872 \div 8,722)\cdot 10^{-6} \text{ m}^2/\text{s} \qquad (11)$$

for n = 100 ÷ 1000 rpm. In the same range of n, but with lower values for ν, the turbulent regime is obtained.

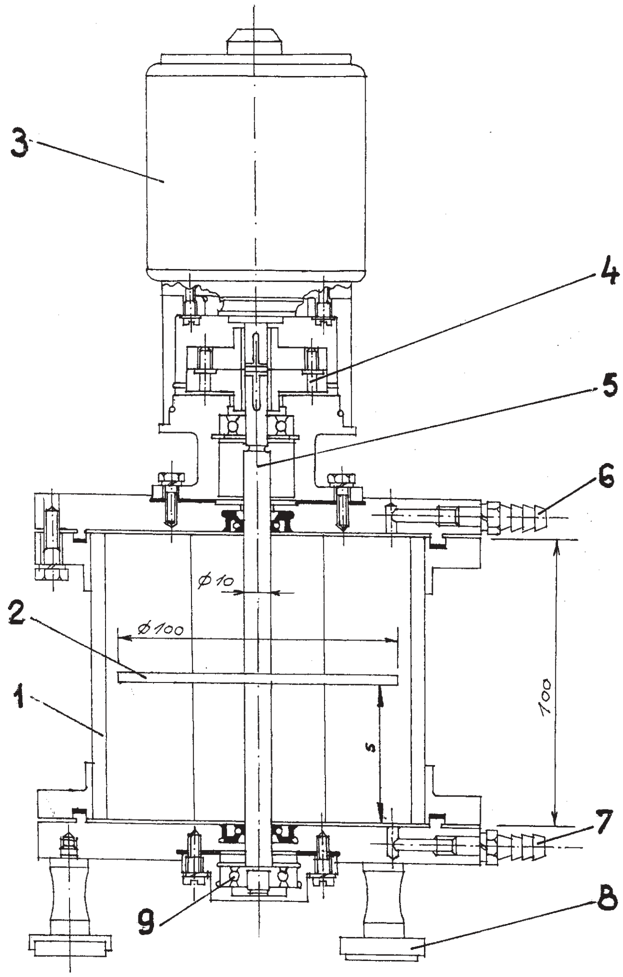

Fig. 2. Experimental unit. 1 – case; 2 – disk; 3 – electrical motor; 4 – elastic coupling; 5 – shaft; 6 – additional pipe for the supply of working liquid; 7 – additional discharge pipe; 8 – plate; 9 – bearing.

Densities and viscosities of the work liquids for laminar regime for a given temperature are presented in tables 1 and 2a ÷ 2d.

Results and discussions

The values obtained during experiments of C^e_M for Re = 100 ÷ 3·10⁴ are given in table 4. Theoretical values of C^t_M were also computed using the widely used expression (4).

Table 1

DENSITY OF WORK FLUIDS FOR LAMINAR REGIME [KG/M³] DEPENDING ON TEMPERATURE [°C]

Liquid / t [°C]	18	24	34	40	45	50	65	72	80
AIR3514	935,4				912,4		892,3		
H 42	899,5			872,5					841,2
SR 11	896,5		884,2					851,6	
MK 8		875,3				850,4			825,1

Table 2

VISCOSITY OF WORK FLUIDS FOR LAMINAR REGIME (ENGLER DEGREES AND m²/s) DEPENDING ON TEMPERATURE

a) OIL AIR 3514

t [°C]	18	19	33	40	45	70
°E	2,178	2,136	2,017	1,711	1,629	1,246
$\upsilon\cdot 10^6$	13,045	12,681	11,636	8,836	8,050	4,056

Table 2
CONTINUARE

b) OIL MK 8

t [°C]	17	28	33	40	46	66
°E	1,922	1,825	1,596	1,394	1,264	1,015
$\upsilon \cdot 10^6$	10,786	9,901	7,729	5,677	1,260	1,213

c) OIL H 42

t [°C]	19	26	34	40	45	70
°E	13,744	7,806	6,599	4,145	3,511	1,759
$\upsilon \cdot 10^6$	100,146	56,331	46,392	28,819	25,700	9,288

d) OIL SR 11

t [°C]	17	30	38	45	57	70
°E	50,567	19,112	11,555	8,537	5,083	3,356
$\upsilon \cdot 10^6$	370,020	139,890	84,582	61,751	35,962	22,685

In order to study an increased interval of Re numbers and to obtain the turbulent regime, the work liquids from table 3 were used.

Table 3
DENSITY AND VISCOSITY OF THE WORK LIQUIDS FOR TURBULENT REGIME AT 18°C

Liquid	ρ [kg/m^3]	$10^3\eta$ [Pas]	υ [10^6 m^2/s]
Water	998,6	1,00	1,00
Medicinal alcohol	908,31	2,48	2,73
Gas	809,15	0,560	6,92

Table 4
DEPENDENCY C_M=f(Re) AT Re = 100 ÷ 3·10^4

Re	109	535	650	915	1228	1581	2195
C_M^e	0,2500	0,1150	0,1073	0,0882	0,0761	0,0671	0,0569
C_M^t	0,3696	0,1672	0,1516	0,1279	0,1104	0,0973	0,0826
C_M^t / C_M^e	1,47	1,45	1,41	1,45	1,45	1,45	1,45
Re	2509	4212	10736	14148	20470	24271	28154
C_M^e	0,0532	0,0411	0,0287	0,0204	0,0186	0,0171	0,0159
C_M^t	0,0772	0,0596	0,0373	0,0325	0,0270	0,0248	0,0236
C_M^t / C_M^e	1,45	1,45	1,45	1,45	1,45	1,45	1,45

If expressions (4) and (8) are compared, it can be seen for the free disk that the values of C_M are 44,9% reduced, which concludes that for cased disks, even at ratios (*s*/*R* = 1) the relation (8) is valid most of the time.

Work liquids presented in table 5 were used for Re numbers > 10^5. The C^t_m values have been computed using (5) for Re < 3·10^5 and using (7) for increased Re numbers.

Table 5
DEPENDENCY C_M(Re) AT Re = (1 ÷ 5) ·10^5

10^{-5}Re	1,166	1,333	2,565	2,988	3,036	3,036	3,688	4,34	5,144
$100 C_M^e$	0,980	0,889	1,942	0,970	0,997	1,896	1,054	1,488	1,355
$100 C_M^t$	1,135	1,059	0,764	0,707	1,170	1,160	1,123	1,09	1,05
C_M^t / C_M^e	1,19	1,16	1,23	1,37	1,17	1,30	1,065	0,732	0,775

The results show that, at Re = 3·10^5, the theoretical values are in average 20% higher, without having a monotone dependency regarding Re, situation met also in other experimental research. If (8) is used, the determined values are with 1,21 ÷ 1,78 higher. A dispersion of results, similar with the one presented in figure 2, is obtained around Re = 3·10^5. At Re > 3·10^5, the C_M^t values have been calculated with (7) and they were smaller than the experimental ones. As a matter of fact, we have to mention that relations (6) and (7) have very closed outputs.

Conclusions

The results obtained for the viscous friction coefficient in the case of Reynolds numbers Re = 10^2 ÷ 10^3 are original.

The values of the viscous friction coefficient measured in the case of laminar flow regime are 45% lower than the theoretical ones (table 4).

In the case of the turbulent flow regime the values of the viscous friction coefficient are 25% lower than the theoretical ones (table 5).

Contrary to an affirmation [6, pp. E151], the influence of the case is significant for high *s*/*R* ratios (*s*/*R* = 1).

Notation list

C_M – Friction moment on disk coefficient
M – Friction moment on disk
s – Distance between disk and case
Re – Reynolds number
ν– Cinematic viscosity
ω – *Angular speed*
ρ - Density

References

1. FLOREA, M., The viscous friction coefficient at disks in uniform rotation (in Romanian), Papers of the 4th conference of Hydro-electricians from Romania, May 2006, V. I., p. 1-8, Bucharest
2. STAVA, P., ZAPONEL, J., WAGNEROVA, ZUBIK, P., Investigation of instabilities during transition from laminar to turbulent flow in the gap between two concentric discs, one rotating, by methods of nonlinear dynamics, Strojnicky casopis, 55, C. 2, 2004, p. 100
3. SCHLICHTING, H., GERTEN, K., Boundary layer theory, 8th Revised and Enlarged Edition, Springer, Bochum, may, 1999
4. SCHLICHTING, H., Grenzschite Theorie, Braun Verlag Karlsrube, 5 Auflage, 1982
5. KURAKAWA, I., IAMADA, T., MATSUMATO, K., Performances of Centrifugal Imppeler of Very Low Specific Speed, Proc. 19th, Symp. on Hydraulic Machinery and Cavitation, V. 2, Singapore, 1998
6. HÜTTE, The engineer’s manual. Fundamentals (in Romanian), Ed. Tehnicã, Bucharest, 1995
7. DEBUCHY, R., Ecoulement turbulent avec aspiration radiale entre un disque fixe et un disque tournant, These de doctorat USTL, Janvier, 1993
8. CAZACU, M.D., PETCU, M., BANESCU, A., Axisymmetrical motion of the viscous liquid in the disk mill, Rev. Roum. Sci. Techn. – Mec. Appl., Tome 33, nr. 2,1988, p. 159
9. CONSTANTINESCU, V.N., The dynamics of incompressible viscous fluids (in Romanian), Ed. Academiei, Bucharest, 1987

Intrat în redacþie: 7.03.2007